

# SU(3) Gauge Theory with Adjoint Fermions

F. Karsch, M. Lütgemeier \*

Faculty of Physics, University of Bielefeld, P.O. Box 100131, 33501 Bielefeld, Germany

We analyze the finite temperature phase diagram of QCD with fermions in the adjoint representation. The simulations performed with four dynamical Majorana fermions, which is equivalent to two Dirac fermions, show that the deconfinement and chiral phase transitions occur at two distinct temperatures,  $T_{\text{chiral}} \simeq 6.65 T_{\text{deconf}}$ . While the deconfinement transition is first order we find evidence for a continuous chiral transition. We also present potentials for  $T < T_{\text{deconf}}$  and  $T_{\text{deconf}} < T < T_{\text{chiral}}$  both for fundamental and adjoint fermion-antifermion pairs.

## 1. Motivation

In QCD the interplay between confinement and chiral symmetry restoration is one of the most puzzling problems. As the two phenomena originate from different non-perturbative mechanisms it has been speculated that QCD would undergo two distinct phase transitions [1]. However, so far all lattice calculations have shown that there is just one unique critical temperature, and the critical behaviour close to  $T_c$  is influenced by chiral as well as confinement properties of the system.

Thus, it is of interest to study an  $SU(N)$  gauge theory with quarks in the adjoint representation, where the two transitions fall apart. This theory has a global  $Z(N)$  center symmetry and an  $SU(2n_f)$  chiral symmetry [2]. Investigations of  $SU(2)$  [3,4] have shown indeed, that the two transitions are well separated with  $T_{\text{deconf}} < T_{\text{chiral}}$ .

In this work we simulate an  $SU(3)$  gauge theory with the usual plaquette action and 2 dynamical staggered (Dirac) fermions in the adjoint representation on  $8^3 \times 4$  and  $16^3 \times 4$  lattices using the exact hybrid  $\Phi$  pseudo fermion algorithm. We ran at up to 15  $\beta$  values in the interval [5.2; 7.0] with a mass between 0.10 and 0.01 (small lattice) resp.  $m = 0.02$  (larger lattice).

We note that our model is closely related to super-symmetric gauge theories (see [5]).

## 2. Phase Structure

In order to clarify the phase structure of our model we analyze the Polyakov loop and the chiral condensate, which are the order parameters for [de-]confinement resp. the chiral transition. On the smaller lattice, which we only discuss in this section, we simulated with several mass values allowing us to extrapolate to the zero mass case.

### Polyakov loop

In figure 1 one sees that the Polyakov loop shows a first order signal around  $\beta = 5.3$ , which is confirmed by a double peak structure of the combined plaquette histogram

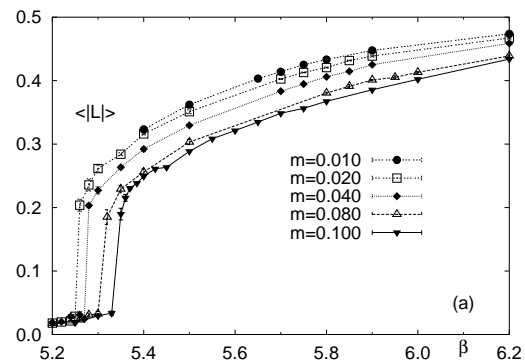


Figure 1. Polyakov loop for several mass and  $\beta$  values on the  $8^3 \times 4$  lattice

In the following table the critical couplings are summarized, which are obtained from the re-weighted Polyakov loop susceptibilities.

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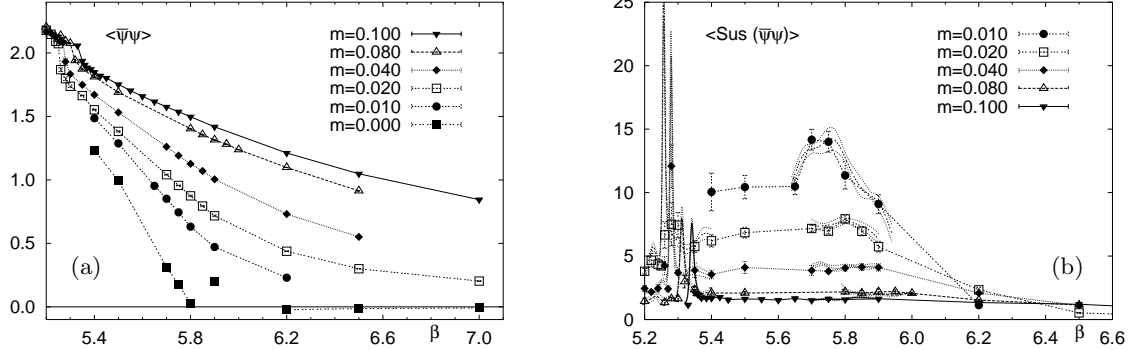


Figure 2. Chiral condensate (a) with  $m \rightarrow 0$  extrapolation (filled squares) and its susceptibility (b) on the  $8^3 \times 4$  lattice

mass	0.10	0.08	0.04	0.02
$\beta_{\text{deconf}}$	5.342(2)	5.312(2)	5.279(2)	5.256(2)

They can be fitted to a linear function with  $\beta_{\text{critical}} = 5.236(3)$  for  $m = 0$ .

### Chiral condensate

For the chiral condensate we must go to the zero mass limit to find the critical coupling, where it drops to zero. This extrapolation of  $\bar{\psi}\psi$  to  $m = 0$  is done with the following fit ansatz.

$$\langle \bar{\psi}\psi \rangle(\beta, m) = a_0 + \bar{a}_1 m^{1/\delta} + a_1 m + a_2 m^2 + \dots (1)$$

For  $\beta \leq 5.8$  a nonzero  $a_0$  and a leading  $\sqrt{m}$ -term is found, above we find a polynomial with  $a_0 \approx 0$ . In figure 2a the finite mass data for  $\bar{\psi}\psi$  and the extrapolated curve is shown. From the latter we can read of  $\beta_{\text{chiral}} \approx 5.8$ , which we can check by looking at the re-weighted chiral susceptibility (fig. 2b).

The first peak around  $\beta = 5.3$  coincides with the deconfinement transition, then in  $[5.4, 5.7]$  the chiral susceptibility stays constant. The values of this plateau is mass dependent and consistent with  $1/\sqrt{m}$ , the  $m$ -derivative of equation 1.

Around  $\beta = 5.8$  there is (for  $m \leq 0.02$ ) a second peak indicating the chiral phase transition. For  $m = 0.02$  we get  $\Delta\beta = 0.544(32)$  and using the two-loop  $\beta$ -function we find  $T_{\text{chiral}}/T_{\text{deconf}} \approx 6.65 \pm 1.13$ . In  $SU(2)$  a qualitative similar behaviour was found with a ratio of  $175 \pm 50$  [3].

The analysis of the larger lattice at  $m = 0.02$  confirms our observations, large finite size effects seem to be absent. In summary our model contains a mixed phase (roughly in  $[5.3; 5.8]$ ), where the quarks are already deconfined but with the chiral symmetry still broken.

### 3. Potentials from Polyakov Loop Correlations

Correlations of fundamental as well as adjoint Polyakov loops are used to get the potential of a static quark-antiquark pair. The potentials for two couplings (5.25, 5.40) are shown in figure 3.

At  $\beta = 5.25$ , i.e. below the deconfinement transition, the fundamental potential can be fitted to

$$V(R) = V_0 + \sigma R - \alpha/R \quad (2)$$

with  $\sigma a^2 = 0.317(7)$  and  $\alpha = 0.198(5)$  while the adjoint potential shows a string-breaking behaviour. At distances larger than 2.2 it is consistent with the constant  $V_{\infty} a = 0.270(1)$ .

At  $\beta = 5.40$  both (normalized) potentials can be fitted to

$$V(R) = -\alpha/R \exp(-\mu R) \quad (3)$$

with these results:

	$\alpha$	$\mu a$
fundamental	0.11 (4)	1.59 (15)
adjoint	0.28 (9)	1.39 (14)

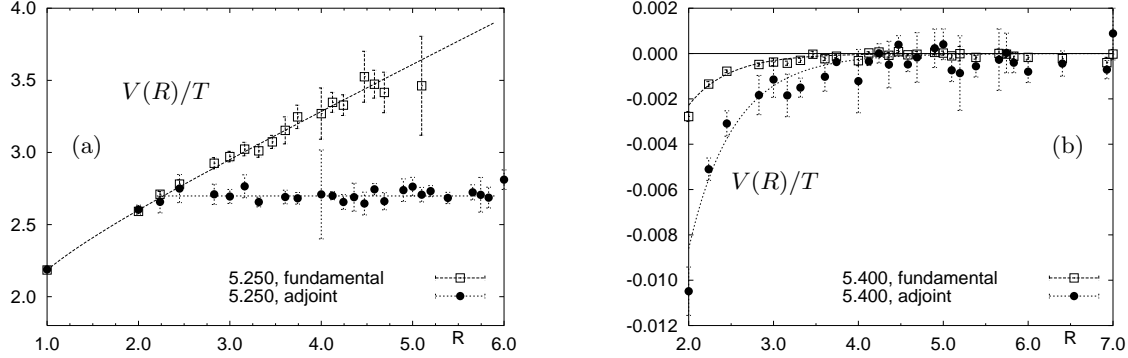


Figure 3. Potentials in the confined phase (a) and the mixed phase (b) on the  $16^3 \times 4$  lattice

#### 4. Thermodynamics around $T_{\text{deconf}}$

To calculate the pressure the  $\beta$ -function and plaquette data for  $T = 0$  are needed ([6,7]). Without it we can only speculate about the general behaviour of the pressure.

The plaquette data show a sharp rise at  $\beta_{\text{deconf}}$  but behave moderately around  $\beta_{\text{chiral}}$  (see figures in [9]). Therefore the pressure (the integrated plaquette) will also rise at  $T_{\text{deconf}}$  but will probably not feel the chiral symmetry restoration. The latent heat can be derived from the gap in the plaquette and the  $\beta$ -function. The former is estimated from the re-weighted plaquette while the latter is known here only perturbatively.

$$\frac{\Delta\epsilon}{T^4} = \frac{\Delta(\epsilon - 3p)}{T^4} = -a \frac{d\beta}{da} \cdot N_\tau^4 \cdot 6 \Delta\text{Plaq} \quad (4)$$

Results are shown in figure 4. For  $m = 0$  we finally estimate

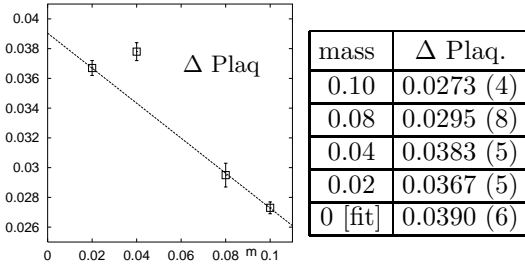


Figure 4. Estimated plaquette differences at  $\beta_{\text{deconf}}$  and their extrapolation to  $m = 0$

$$\frac{\Delta\epsilon}{T^4} = 14.4 = 0.36 \cdot \frac{\epsilon_{SB}(N_\tau = 4)}{T^4} \quad (5)$$

This result is very close to the pure gauge case, where the latent heat is about 34% of the Stefan-Boltzmann limit at  $N_\tau = 4$  [8]. On the other hand the latent heat per degree of freedom in the deconfined phase is much lower, because there are now 64 fermionic degrees of freedom instead of 24.

#### 5. Conclusions

Analyzing  $SU(3)$  with 2 adjoint fermions we find two distinct phase transitions with  $T_{\text{chiral}}/T_{\text{deconf}} \approx 6.65$ . In the intermediate phase (deconfined but chirally broken) the chiral condensate depends on  $\sqrt{m}$ . The deconfinement transition seems to be first order with a latent heat of 35% of the Stefan-Boltzmann limit while the chiral transition is continuous. We also note that it may be interesting to study this model at non-zero chemical potential as its fermion determinant stays real and positive in this case.

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